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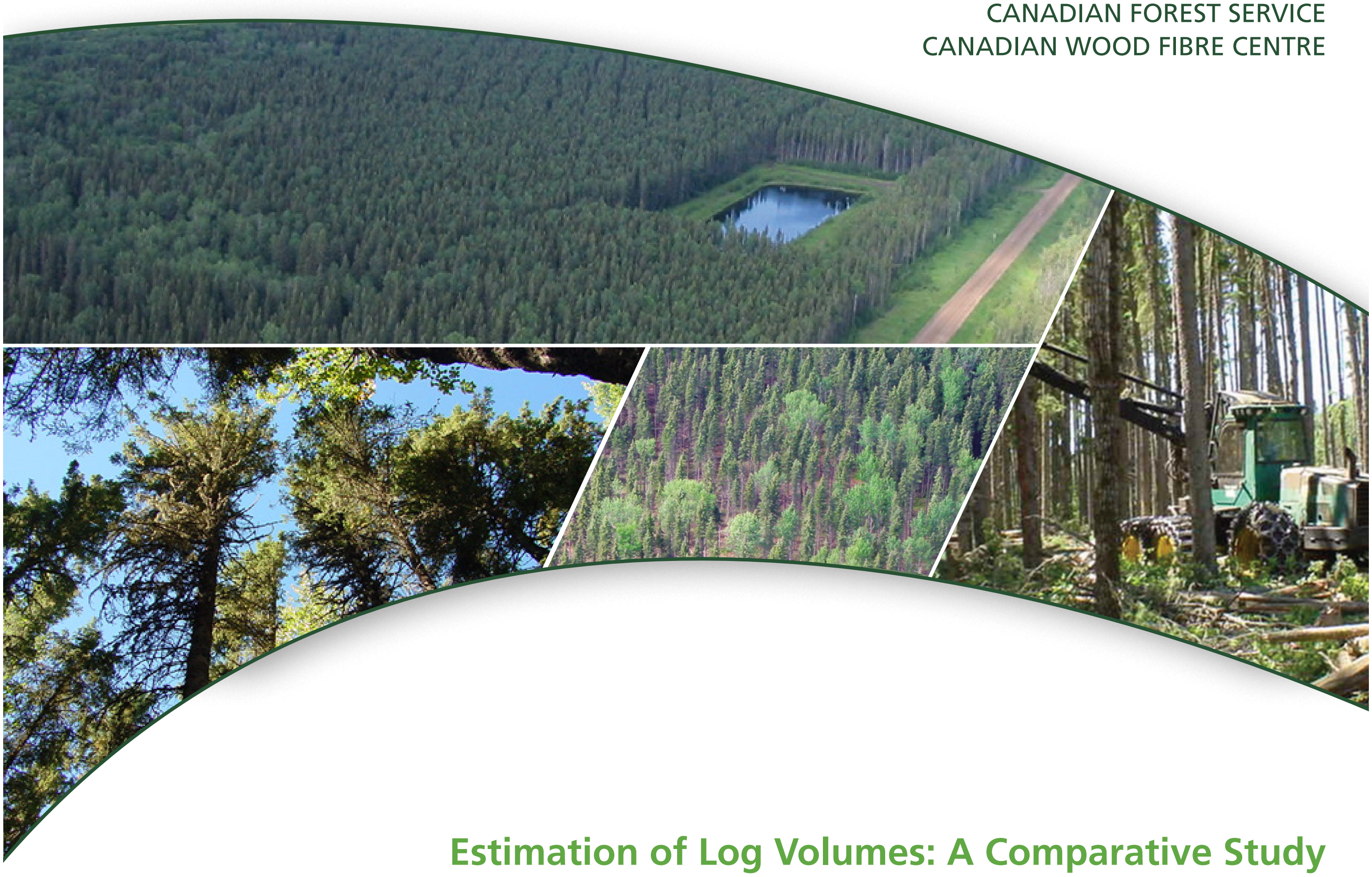

Estimation of Log Volumes: A Comparative Study

C. Li, H. Barclay, H. Hans, and D. Sidders

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FI-X-11

2015

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Estimation of Log Volumes: A Comparative Study

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Abstract

Log volume estimation is a central topic in forest science research and forestry practice because accurate estimates are essential for commercial harvesting, sustainable forest management, and conservation. To determine whether the log volume estimates obtained using different commonly used log volume models are consistent, we reviewed major log volume estimation methods and conducted a comparative study using data from six different treatments of a commercial thinning experiment in the Boreal Plains Mixedwood Fibre Initiative project. There were significant differences in the log volume estimates we obtained using the different methods under simulated tree-length harvesting conditions. However, these differences were reduced when the section length of a log was decreased and the stem volume was calculated as the sum of all sections under simulated cut-to-length harvesting conditions, and only minor differences were observed when the section length was decreased below a certain threshold section length.

Résumé

L’estimation du volume de bois marchand est un élément important de la recherche scientifique en foresterie et des pratiques de l’industrie forestière, du fait que des estimations précises sont essentielles pour l’exploitation commerciale, la gestion durable des forêts et la conservation. Pour déterminer si les estimations de volume de bois marchand obtenues au moyen de différents modèles couramment utilisés sont cohérentes, nous avons examiné les principales méthodes d’estimation de volume de bois marchand et mené une étude comparative en utilisant les données obtenues lors de six différents traitements d’une expérience d’éclaircie commerciale effectuée dans le cadre de l’Initiative de la fibre de bois des plaines boréales à forêts mixtes. Des différences significatives ont été notées dans les estimations de volume de bois marchand que nous avons obtenues avec les différentes méthodes dans des conditions simulées de récolte d’arbres de longueur variée. Cependant, ces différences ont diminué lorsque la longueur d’une section de bois marchand était moindre et que le volume du tronc était calculé comme la somme de toutes les sections dans des conditions simulées de récolte de bois coupé. Par ailleurs, des différences mineures ont été observées lorsque la longueur de la section a été réduite sous un certain seuil de longueur.

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1. Introduction

Wood volume estimation has been a central research topic in forest science, because accurate estimates of wood volume are essential in sustainable forest management and for trade in forest resources (Davis et al. 2001). Understanding the volume of wood in forests and regions is fundamental for regional forest management planning, commercial harvest, and conservation. As well, jurisdictions are increasingly estimating the overall volume of their forest inventories, and these volume estimates will be valuable in the modeling of carbon budgets. The accuracy of wood volume estimation could influence sustainable forest management planning and decision-making on wood utilization (Leuschner 1984). In market trade, better estimates of wood volumes could help ensure fair trades between sellers and buyers. However, the use of different formulas and models often causes confusion, as does the fact that there are different approaches and methods for estimating wood volume. As a consequence, it can be a challenge for forest managers and practitioners to find reliable methods of wood volume estimation that suit their purposes.

Wood volume is a cubic measure of the amount of usable wood present in an individual log, tree or group of trees and is used to assess economic value (FAO 1997). Wood volume is generally estimated on the basis of the stem wood of standing softwood trees; branches may be included for hardwood tree species. From the perspective of the forest value chain, forest wood volume may be referred to as (1) *standing wood volume*, expressed either as *stem volume* or as *merchantable volume* (a stem volume that has been truncated according to a given utilization standard); (2) *log volume*, which is the merchantable volume that arrives at a mill's gates, reduced as a result of losses during harvest and transportation; or (3) *product volume*, which is the amount that can be sold in the market (including lumber, chips, sawdust, shavings, barks, plywood and veneer sheets, hog fuel for bioenergy, and chemical products), reduced as a result of mill operations. Figure 1 shows the relationships among these volumes and their influencing factors.

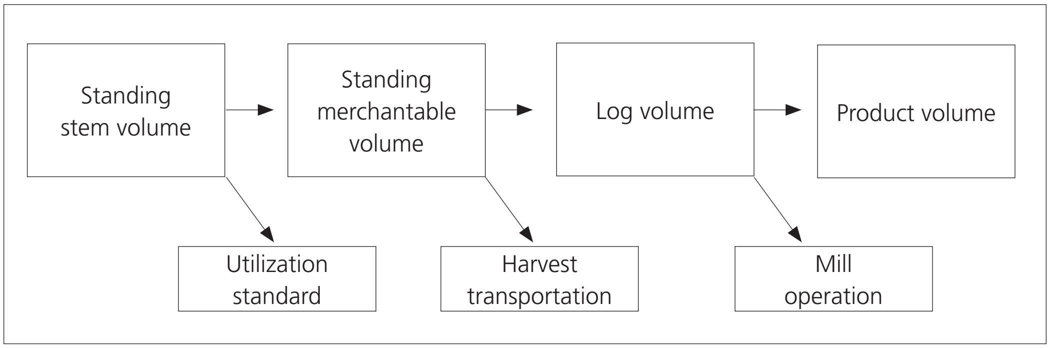

Figure 1. Various wood volumes in a forest wood value chain.

These various wood volumes are used for different purposes. Standing wood volumes (both total gross and merchantable) are often used in forest management planning. These volumes are obtained from forest inventory data and the estimation models are based on sampling data from a small subsample of forest stands. Log volume, usually used in mill operations to estimate the wood volume arriving at the mill's gates, can be obtained from direct measurements or from a scan of log dimensions (Janák 2012) or log weight or both. Product volumes are the output from mill processing and are commonly used in commercial markets. This paper focuses on log volume estimation.

Various methods of estimating log volumes are documented in the literature and widely applied in forest science research and forestry practice. It is commonly assumed that these different log volume models or methods yield equivalent estimates of log volume. If this were true, any method could be considered as applicable as any other, and there would be no need for concern about the accuracy of the estimation. However, this assumption has not been examined in a systematic manner.

The objectives of this paper are (1) to review the major methods of log volume estimation and (2) to compare the accuracy of these methods. The results of this comparative study can assist forest managers and practitioners to improve their planning by using the most accurate estimation tool.

2. Materials and Methods

Log measurements are usually made on clean trunks, that is, main stems from which the branches were removed after the trees were felled. The stems are usually crosscut to standard lengths and/or to a top diameter determined by market demand. Consequently, the main concerns in this study were the accuracy of log measurement and the choice of method for estimating the volume of these log lengths.

2.1. Methods of Log Volume Estimation

Methods of log volume estimation can be categorized into three groups: log volumes can be estimated on the basis of (1) direct log measurement, (2) tree taper models, and (3) scanned log dimensions.

2.1.1. Log Volume Estimation from Direct Log Measurement

Methods that use log volume estimation are classic approaches that can be traced back a very long time. They were developed in the era before computer technology became available and are based on assumed geometric solids of wood logs. They are well documented in forestry handbooks (e.g., Wenger 1984), textbooks (e.g., Briggs 1994), and other professional materials (e.g., Avery and Burkhart 1994), and they are still widely used in forestry practice.

Log volume can be expressed in either imperial (cubic feet) or metric (cubic metres) units, and there are three different ways to estimate it: by measurement of (1) individual log dimensions, (2) log weight, or (3) cord measure. Log weight measurement is used mainly for relatively small trees and logs, which are paid for by the metric ton in markets. This measurement is unsuitable for estimating the volume of larger logs because weight per unit volume varies by tree species (due to their differing wood densities), by time of year (because the season affects moisture content), by tree age, and by other factors. Cords are defined as stacks of wood that occupy a volume of 128 feet^3 (or 3.6246 m^3). Cord measure is usually used for wood stacks at the roadside and is determined by calculating the width, length, and height of the stack. For this method to be accurate, all logs in a stack must be of uniform length, and the stack needs to be built neatly and tidily. Because of the restricted application of these methods, the current study focuses on the first estimation method: measurement of individual log dimensions.

Log scaling rules have been developed based on the measurements of the diameter of the small end, midlength, and large end (*SED*, *MED*, and *LED*, respectively) and the length of the log (*LL*) for each harvested log (Wenger 1984; Briggs 1994; Avery and Burkhart 1994; Janák 2012). The most commonly used cubic volume formulas for merchantable volume V_m are listed in Table 1.

Table 1. Commonly used volume formulas for harvested wood stems (Briggs 1994).

Source of formula	Equation	Equation number
Smalian	$V_m = ƒ(SED^2 + LED^2)\ LL/2$	(1)
Bruce	$V_m = ƒ(0.75SED^2 + 0.25LED^2)LL$	(2)
Huber	$V_m = fMED^2LL$	(3)
Newton	$V_m = ƒ(SED^2 + 4MED + LED^2)LL/6$	(4)

In equations, ƒ = 0.00007854 for metric units (cubic metres) and ƒ = 0.005454 for imperial units (cubic feet). These two ƒ values provide conversions in the two-unit systems; thus, the volume calculated in one unit can be converted to the other.

These formulas were derived from the geometry assumption that also implicitly assumes no taper and sweep were present, rarely true in reality, and that the logs are circular in cross-section; thus, the log volumes can be calculated as they are for geometric solids. The lower, middle, and upper portions of a tree stem are approximated as a truncated neiloid, a truncated paraboloid, and a truncated conoid, respectively (e.g., Wenger 1984).

It is well established that the volume estimates from various formulas do not provide exactly the same results. Each can have a bias compared with the true volume because of the discrepancy between the assumed geometric shape and the actual log shape (Janák 2012). For example, Briggs (1994) pointed out that Smalian's formula (Eq 1) usually overestimates log volume because it assumes a paraboloid log shape. Bruce's butt log formula (Eq 2) is a variation of Smalian's formula that accounts for the changes in the butt portion of the log by altering the weights of *SED* and *LED* in the calculation. Huber's formula (Eq 3) assumes the average cross-section area is at the midlength of the log, although this is not always true. Huber's formula has been considered to be of limited use in North America because of the impracticality of *MED* measurement; for instance, it takes a longer time to determine exactly where the point of midlength is. However, it is widely used in Central Europe in conjunction with scan technology (Janák 2012). Newton's formula (Eq 4) has been recognized as the most accurate; however, it requires measurements of *SED*, *MED*, and *LED*. Smalian's formula (Eq 1) is the most widely used in forestry practice.

2.1.2. Log Volume Estimation from Forest Growth and Yield Models

When wood volume lost as a result of harvest and transportation is known, the log volume can also be obtained from forest volume models or yield tables. These models have been widely used in forestry practice and forest management planning. They can be expressed at the level of either the individual tree or the stand. At the individual tree level, they predict total stem volumes (V) as a function of diameter at breast height (DBH) and total tree height (H). Field measurements of DBH and H of individual trees can be used to model V for given tree species in a given region. Two examples of V models follow, one from Honer et al. (1983) and the other from Penner et al. (1997).

Honer et al. (1983) provided timber tables for 21 major commercial tree species in central and eastern Canada. The V model expresses the volume-diameter-height relationship. For example, for white spruce (*Picea glauca* [Moench] Voss.), V is calculated as follows for 2692 trees from New Brunswick, Quebec, Ontario, Manitoba, Saskatchewan, and Alberta:

$$V = 0.0043891 DBH^2(1 - 0.04365 b_2)^2 / (c_1 + (0.3048 c_2 / H)) \qquad (5)$$

where $b_2 = 0.176$, $c_1 = 1.440$, and $c_2 = 342.175$.

Penner et al. (1997) published species-specific volume models for forest inventory purposes for regions across Canada. For example, the following models estimate V of white spruce in west-central Canada:

Alberta:

$$V = 4.328336 \times 10^{-5}\ DBH^{1.882751} H^{1.02411} \qquad (6)$$

Manitoba:

$$V = -1.331 \times 10^{-3} + 3.292128 \times 10^{-3} (DBH^2 H)/100 \qquad (7)$$

Northwest Territories:

$$V = 4.316 \times 10^{-2} + 3.1526 \times 10^{-5}\ DBH^2 H \qquad (8)$$

The V_m is a truncated whole stem volume V calculated according to a given utilization standard, which consists of stump height h_s (butt end of harvested tree stem) and merchantable height h_m (top end of harvested tree stem), which can differ regionally. For example, in Manitoba, the h_s is 15 cm and h_m is the height at which the diameter inside bark at the top end of the usable stem (DIB_{top}) is 7.62 cm. In Alberta, the h_s is 30 cm and the most commonly used DIB_{top} values are 5 cm, 7 cm, 10 cm, 11 cm, 13 cm, and 15 cm (Huang 1994), depending on operational considerations and/or target utilizations.

Honer et al. (1983) developed a relationship between V and V_m according to utilization standards. For white spruce, the V_m is

$$V_m = V(r_1 + r_2 X_3 + r_3 X_3^{\ 2}) \qquad (9)$$

where $X_3 = DIB_{top}^{\ 2} / (DBH^2(1 - 0.04365 b_2)^2)(1 + h_s / H)$, $b_2 = 0.176$, $r_1 = 0.0236$, $r_2 = 2.2191$, and $r_3 = -1.2705$.

Consequently, V_m can be estimated from the V estimates of any volume model, such as models of

V estimation developed for Canada's National Forest Inventory (Penner et al. 1997).

The *V* obtained at the individual tree level can be scaled up to stand level, at which the models predict stem volume per unit area as a function of forest stand age, as well as of site index and tree densities in some cases. A simple and commonly used method is to calculate *V* and V_m from measurements at the plot level and scale them up using the plot area to values per hectare.

2.1.3. Log Volume Estimation using Tree Taper Models

Tree taper reflects the geometric shapes of trees. Taper models are derived from the results of stem analysis, based on diameter measurements at stump height, at breast height (1.3 m), and at regular intervals upwards to the top of a tree. Taper models are usually region dependent and species specific; they may involve various mathematical forms for predicting the diameter or *DIB* at any height along the stem.

Two general approaches have been used to develop tree taper models. Earlier research efforts tended to consider various sections of the tree separately, developing a unique function for each section and connecting these functions smoothly at each join point (Ormerod 1973; Demaerschalk and Kozak 1977). The other approach is to develop a continuous function describing the profile of the tree stem along its entire height (Kozak 1988; Newnham 1988). Table 2 lists some examples of these two kinds of taper equations.

Table 2. Examples of sectional and continuous taper functions describing the profile of the tree stem.

Model	Equation	Equation No.
Ormerod's (1973) sectional model	$DIB_i = (D_i - C_i)[(H_i - h)/(H_i - k)]p_i + C_i, p_i > 0$ where DIB_i is the diameter inside bark at h_i in centimetres; H_i is the total height of the ith section in metres; D_i is the measured diameter at height k in centimetres; C_i is the ith section diameter intercept; and p_i is the fixed exponent of the ith section	(10)
Max and Burkhart's (1976) polynomial model	$(DIB_i / DBH)^2 = b_1(Z_i - 1) + b_2(Z_i^2 - 1) + b_3(a_1 - Z_i)^2 I_1 + b_4(a_2 - Z_i)^2 I_2$ where a_1 is the upper join point; a_2 is the lower join point; $Z_i = h_i / H$; H is the total height in m; $I_1 = 1$ if $a_1 - Z_i \geq 0$, $I_1 = 0$ if $a_1 - Z_i < 0$, $I_2 = 1$ if $a_2 - Z_i \geq 0$, and $I_2 = 0$ if $a_2 - Z_i < 0$; and b_1 to b_4 are regression coefficients	(11)
Kozak's (1988) model	$DIB_i = a_0 DBH^{a1} a_2^{DBH} X_i^{b_1 Z_i^2 + b_2 \ln(Z_i + 0.001) + b_3 \sqrt{Z_i} + b_4 e^{Z}i + b_5 (DBH / H)}$ where h_i is the height above the ground in metres, with $0 \leq h_i \leq H$; $Z_i = h_i / H$; $X_i = (1 - \sqrt{h_i / H}) / (1 - \sqrt{p})$; $p = (HI/H) \times 100$; HI is the height of the inflection point, commonly taken to be 20% to 25% of the total height, for example, 22.5% is used in Alberta (Huang 1994); DBH is diameter outside bark at breast height (i.e., 1.3 m) in centimetres; and a_0, a_1, a_2, b_1, b_2, b_3, b_4, and b_5 are parameters	(12)
Newnham's (1988) model	$(DIB_i / DBH)^k = (H - h_i) / (H - 1.3)$ where $k = a_0 + a_1 x_i^6 + a_2(DBH / H) + a_3 x_i^2 (DBH / H)$, and $x_i = (H - h_i) / (H - 1.3)$	(13)
Bi's (2000) trigonometric model	$DIB_i = [\ln \sin(Z_i \pi / 2) / \ln \sin (B\pi / 2)]^k$ where $B = 1.3 / H$, and $K = a_1 + a_2 \sin(Z_i \pi / 2) + a_3 \cos (3Z_i \pi / 2) + a_4 \sin (Z_i \pi / 2) / Z_i + a_5 DBH + a_6 Z_i \sqrt{DBH} + a_7 Z_i \sqrt{H}$	(14)
Sharma and Zhang's (2004) model	$(DIB_i / DBH)^2 = b_0 (h/1.3)^{2-(b_1 + b_2 Z_i + b_3 Z_i^2)} [(H - h)/(H - 1.3)]$	(15)

Kozak's taper model (Eq 12), known as the variable-exponent taper equation, has been further developed into three other forms (1994, 2001, and 2002) (Kozak 2004) to improve its performance; the original model was adopted by the British Columbia Ministry of Forests in 1989 (Kozak 2004) and is also being used in Alberta (Huang 1994; Huang et al. 1999), Saskatchewan (Gal and Bella 1994), Manitoba (Klos et al. 2007), and several regions of the United States (Li et al. 2012), Europe (Hjelm 2011), and Asia (Wang et al. 2007). As a result, the use of Kozak's taper model (Eq 12) has been commonly reported in the literature over the past couple of decades, which makes it convenient to use for various comparisons and tests.

2.1.4. Log Volume Calculation from Scanned Log Dimension in Sawmills

Given the improvements in computing power and information technology in recent years, scanners in sawmills can now incorporate actual log dimensions and data on visible defects to provide accurate measurements for optimal bucking and cutting patterns to enhance lumber production and reduce costs of processing. Using Optitek (Forintek Canada Corp. 2006; FPInnovations 2014) as an example, a three-dimensional log can be represented by a series of sections along the longitudinal (Z) axis of the log, and each section can be represented by a conic section connected through a certain number of points (usually 60 to 240 points for "true shape").

However, this technology has been generally limited to harvested logs at different stages of processing, mainly in sawmills. It is neither economically possible nor ecologically desirable to estimate volume by scanning all of the trees in a stand or forest for forest management planning. Hence, it is unlikely to be applied to standing forests or trees in the near future.

2.2. Data Source for Comparative Study

Six treatment data sets were collected as part of the Boreal Plains Mixedwood Fibre Initiative project of the Canadian Wood Fibre Centre. The data sets came from a commercial thinning trial of white spruce in northern Alberta. Statistical descriptions of the data sets are presented in Table 3.

Table 3. Summary statistics of the six data sets.

Data set	*DBH* (cm)					*H* (m)				
number	*n*	Mean	Max	Min	SD	Mean	Max	Min	SD	Plot description
1	164	20.3	36.6	7.4	6.8	21. 4	30.6	4.4	4.5	Control pre-harvest
2	197	21.6	36.9	9.4	6.6	22.4	35.6	13.1	4.1	Control 2010
3	246	18.8	39.9	7.3	6.1	20.0	29.0	6.1	3.9	Treated pre-harvest
4	100	22.4	35.4	7.3	5.0	21.7	27.0	6.1	3.3	Treated post-harvest
5	146	16.4	39.9	7.5	5.6	18.8	29.0	8.2	3.9	Treated harvest
6	77	23.1	36.8	7.5	5.0	22.1	27.5	6.2	3.1	Treated 2010

DBH = diameter at breast height; H = total height; SD = standard deviation. Treated data sets are plots with commercial thinning treatment.

The dimensions of potential logs from these trees were determined using a wood utilization standard of 15 cm stump height h_s and 7 cm DIB_{top} and by assuming no loss of volume during harvest and transportation. This method for determining dimensions is appropriate for estimating log volume from existing forest inventory.

2.3. Comparison of Log Volume Estimation under Tree-Length Harvesting Conditions

Log dimensions are determined by the harvest operation and on-site bucking practice. There are three major harvesting methods used in Canada: cut-to-length, tree-length, and full-tree harvesting. The cut-to-length method is mainly used for shortwood (Pulkki 1998), in which trees are felled, delimbed, and bucked from the stump to the lengths required for various uses such as sawlogs, pulpwood, and veneer bolts. Under the tree-length and full-tree harvesting methods, trees are felled and skidded to landing sites, then either transported whole or cut to length according to mill requirements before transport. Consequently, log volume estimation could be performed either on shorter logs under the cut-to-length method or on longer logs under the other two methods.

In this comparison, tree-length logs were used to test the assumption that consistent log volume estimates can be obtained using any of the V_m models. If this were true, log volume estimations using different V_m models should be the same or very similar. In the calculation, *SED* was determined by the utilization standard for DIB_{top}, *LED* by Kozak's tree taper model (Eq 12) at the stump height h_s, *MED* as the average of *SED* and *LED*, and *LL* as merchantable height h_m minus stump height h_s. The parameters used in estimating V_m from Kozak's taper model (Eq 12) for white spruce were $a_0 = 0.860438$, $a_1 = 0.995406$, $a_2 = 0.998493$, $b_1 = 1.040218$, $b_2 = -0.252387$, $b_3 = 1.040218$, $b_4 = -0.852227$, $b_5 = 0.110359$, and $p = 0.225$ (Huang 1994). For convenience of comparison, all V_m estimates were scaled up to 1 ha. The relative accuracy was expressed as the relative percentage bias compared with the estimate from Kozak's taper model:

$$B_i = (V_i - V_{Kozak})/V_{Kozak} \times 100 \tag{16}$$

where B_i is the relative percentage bias of method *i* compared with Kozak's taper model, and V_i and V_{Kozak} are the wood volumes calculated using method *i* and Kozak's taper model, respectively.

2.4. Comparison of Log Volume Estimation under Cut-to-Length Harvesting Conditions

There could be a variety of log lengths under the cut-to-length harvesting method, depending on market demand. For use as sawlogs, for instance, felled trees are crosscut into shorter logs of 8 feet (2.44 m) for stud mills, and into logs ranging from 8 to 16 feet (2.44 m to 4.88 m) for random-length mills. As a result, to assess the accuracy of estimation, comparisons need to be made at different log lengths. These comparisons were carried out by simulating crosscutting the logs into different numbers of sections and calculating V_m as the sum of all the sections of a log. In calculating V_m for each section, Kozak's tree taper model (Eq 12) was employed to determine the *SED* and *LED* of the section, and the *LL* of the section was determined by a series of fixed section lengths. We started with the largest section length of 40 m to ensure that the longest log (35.6 m in our data sets) would be covered and that a single section of the whole log would be simulated. We used 19 other fixed testing section lengths of logs ranging from 30 m to 0.001 m. Such a systematic simulation design allowed us to explore the detailed, changing pattern of relative accuracy using Smalian's, Huber's, Bruce's, and Newton's methods.

3. Results and Discussion

3.1. Log Volume Estimation under Tree-Length Harvesting Conditions

Table 4 shows the V_m estimated using different methods for the six data sets. The estimates differed significantly among the various data sets.

Table 4. Log volume predictions from different methods.

		Log volume (m³/ha)					
Method	**Equation number**	**Data set 1**	**Data set 2**	**Data set 3**	**Data set 4**	**Data set 5**	**Data set 6**
Smalian	1	606.51	523.71	534.00	307.19	226.81	256.40
Bruce	2	340.62	291.83	305.56	171.93	133.62	142.73
Huber	3	399.12	341.52	359.80	204.14	155.66	168.81
Newton	4	468.25	402.25	417.86	238.49	179.38	198.01
Kozak	12	404.95	346.74	364.95	207.15	157.80	171.53
Honer	5, 9	375.27	320.80	342.85	195.07	147.78	161.53
Penner-Honer	6, 9	388.31	332.49	350.39	198.84	151.55	164.71

To facilitate comparison, Eq 16 was used to transform the differences among various testing data sets into percentages (Table 5), in which positive values indicate overestimation and negative values denote underestimation.

Table 5. Bias of various log volume estimation methods relative to the estimate from Kozak's taper model.

	Relative bias (%)						
Method	**Data set 1**	**Data set 2**	**Data set 3**	**Data set 4**	**Data set 5**	**Data set 6**	**Mean**
Smalian	49.77	51.04	46.32	48.29	43.73	49.48	48.11
Bruce	–15.89	–15.84	–16.27	–17.00	–15.32	–16.79	–16.19
Huber	–1.44	–1.51	–1.41	–1.45	–1.36	–1.59	–1.46
Newton	15.63	16.01	14.50	15.13	13.68	15.44	15.06
Honer	–7.33	–7.48	–6.06	–5.83	–6.35	–5.83	–6.48
Penner-Honer	–4.11	–4.11	–3.99	–4.01	–3.96	–3.98	–4.03

The relative bias for each of the methods was relatively consistent among the six testing data sets; thus, the mean relative bias (right-hand column of Table 5) can provide an overall assessment of accuracy in the V_m estimation.

Huber's method provided the V_m estimation with the smallest bias (1.46% underestimation; Table 5), which is consistent with Janák's (2012) method (which gives generally lower volume estimates). The next smallest bias was obtained using a combination of the methods of Penner et al. (1997) and Honer et al. (1983) (4.03% underestimation), followed by Honer et al.'s (1983) method (6.48% underestimation). Intermediate biases (16.19% underestimation and 15.06% overestimation) were obtained from Bruce's and Newton's methods, respectively. Smalian's method produced the largest bias (48.11% overestimation).

These results are consistent with known biases in some of the methods; Briggs (1994) reported that Smalian's method overestimates V_m and that Bruce's method has a smaller bias than Smalian's. Previously believed to be the most accurate method (Briggs 1994), Newton's formula exhibited an intermediate bias in our analysis. Surprisingly, Huber's method, which requires only *MED* measurement for the estimate, provided the smallest bias in V_m.

Newton's formula uses three variables of *LED*, *MED*, and *SED* to estimate log volume; however, the performance of this formula was not superior in this study. This result can probably be attributed to the fact that *MED* was approximated using the average of *LED* and *SED*, which meant that *MED* was actually not an independent variable and thus only *LED* and *SED* were used for volume estimation. The performance of Newton's formula could be improved by estimating *MED* using Kozak's (1988) taper model.

3.2. Log Volume Estimation under Cut-to-Length Harvesting Conditions

Figure 2 shows the simulation results of cumulated V_m using four volume models (Smalian's, Huber's, Bruce's, and Newton's) for different section lengths of a log. The results show that the biased V_m estimates increase with increased log section length, reaching a maximum for section lengths longer than 20 m.

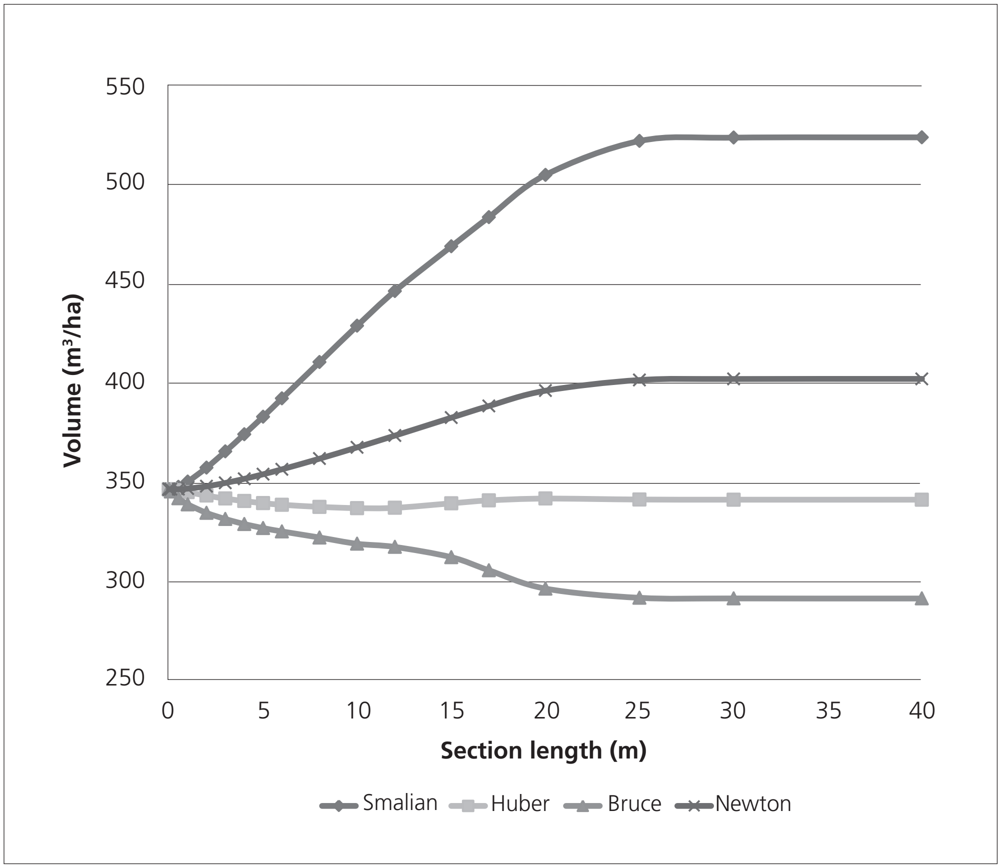

Figure 2. Cumulated log volume estimated using four different methods (Smalian's, Huber's, Bruce's, and Newton's models) for different section lengths of a log.

Under the cut-to-length harvesting method, the *LL* is usually not very long; thus, the relative bias of each V_m method for shorter *LL* needs to be examined. Table 6 shows the V_m estimates and their bias relative to Kozak's taper method (Eq 12) for three selected section *LLs*: the sawlog lengths for stud mills (2.5 m) and for random-length mills (5 m), as well as a 10-m *LL*, simply to show the relative bias when *LL* is doubled.

Table 6. Log volume estimates and their relative biases for selected section lengths.

Section length (m)	Model; log volume estimate (m^3/ha)				Model; relative bias (%)			
	Smalian	Huber	Bruce	Newton	Smalian	Huber	Bruce	Newton
2.5	361.59	342.77	333.22	349.05	4.28	–1.14	–3.90	0.67
5	383.22	339.78	327.25	354.30	10.52	–2.01	–5.62	2.19
10	428.97	337.27	319.33	367.84	23.72	–2.73	–7.91	6.09

For 2.5-m sawlogs, Newton's method provided the most accurate estimate (0.67% overestimation), whereas Smalian's method led to the largest bias (4.28% overestimation), which is still probably within the range of allowable error for forestry practice purposes. For 5-m sawlogs, Huber's method provided the most accurate estimate (2.01% underestimation), whereas Smalian's method still had the largest bias (10.52% overestimation), which might fall slightly outside the range of allowable error. However, for 10-m logs, Smalian's method could lead to a significantly large bias (23.72% overestimation).

4. Conclusions

Significant differences in log volume estimates were obtained from different estimation methods under tree-length harvesting conditions, that is, when each tree stem will only be measured once. However, under cut-to-length harvesting conditions, for log lengths of less than 20 m, these differences were decreased with decreasing section length of the log and when merchantable volume for the tree was calculated as the sum of all of its sections. Furthermore, when the section length is decreased sufficiently, very minor differences would be expected.

In summary, we found the following:

- selection of a suitable model or method of log volume estimation is important because different models or methods could yield different estimates of log volume;
- under tree-length harvesting conditions, Huber's method provided the smallest bias and Smalian's method produced the largest bias;
- under cut-to-length harvesting conditions,
- Newton's method provided the most accurate estimate and Smalian's method led to the largest bias for 2.5-m sawlogs;
- Huber's method provided the most accurate estimate and Smalian's method had the largest bias for 5-m sawlogs;
- Smalian's method could lead to a significantly overestimation for 10-m logs; and
- taper models such as Kozak's model (1988) are recommended for estimating log volumes because they characterize the diameter changes from the butt to the top of trees, so that the log volumes under a flexible section length could be estimated without making assumptions on tree form.

Forest managers and researchers can use the results presented in this report to select a suitable method of log volume estimation. Our findings will also help them understand the bias associated with the estimation method they have selected.

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