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BIOMASS OF THE MERCHANTABLE AND UNMERCHANTABLE PORTIONS OF THE STEM

I.S. Alemdag
Petawawa National Forestry Institute

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Contents

Figures

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Abstract
Equations for estimating separately the mass of merchantable and unmerchantable (stump and top) portions of the stem for variable top diameters and heights were derived for nine tree species. Results are expressed as percentages of ovendry mass of the total stem wood plus bark. Examples of application are provided.

Résumé
Des équations de biomasse ont été établies pour neuf espèces d'arbre. Elles estiment séparément la masse des sections non marchandes (souche et sommet) et marchandes de la tige et tiennent compte de hauteurs variables et de différents diamètres à hauteur marchande. Les résultats sont exprimés en pourcentage de la masse anhydre de la totalité de l'arbre comprenant l'écorce. Des exemples d'application sont également fournis.

INTRODUCTION

Biomass equations have been developed for many tree species from various regions in Canada. With some possible exceptions, these equations produce ovendry mass estimates by tree components, mainly for stem wood, stem bark, live branches and twigs plus leaves (or needles). The whole tree estimate is derived either by the summation of those values or by employing a separate equation. The resulting estimates enable calculation of the gross residues left in the forest or lumber yard after the removal of the desired part of the tree, the stem wood. However, a refinement is necessary, since only the merchantable part of the stem is customarily removed for conversion, and the rest is left unutilized. This unmerchantable top represents a source of potential energy and its amount should be known. The aim of this report is to develop equations for estimating separately the ovendry mass of the merchantable (utilized) portion of the stem, and the top (the remaining part). Another remaining part, the stump, is to be investigated within the merchantable portion. The tree species are: red pine (Pinus resinosa Ait.), jack pine (Pinus banksiana Lamb.), black spruce (Picea mariana (Mill.) B.S.P.), white spruce (Picea glauca (Moench) Voss), balsam fir (Abies balsamea (L.) Mill.), trembling aspen (Populus tremuloides Michx.), largetooth aspen (Populus grandidentata Michx.), white birch (Betula papyrifera Marsh.) and red oak (Quercus rubra L.). Sample material was collected only in Ontario.

PREMISES

Definitions

In the present study, for the purpose of analysis, the merchantable portion of the stem was defined as the part of the stem from ground level to a given merchantable top diameter and having a minimum length of 2.80 m. The merchantable portion was then separated into its two components: wood (hereinafter called

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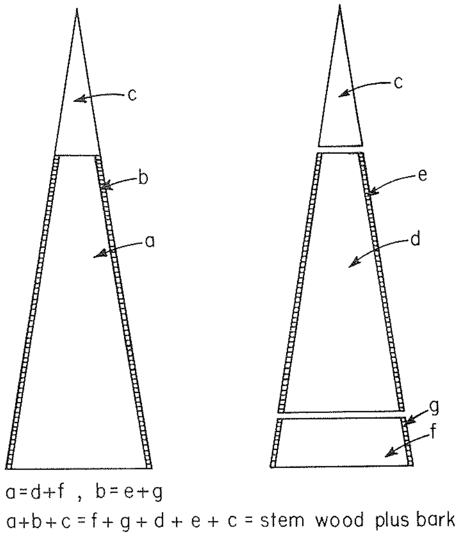

Figure 1. Schematic presentation of the analyzed stem components.

merchantable wood) and bark (hereinafter called merchantable bark). The top of the stem above the merchantable portion was studied without separating it into wood and bark. The stump was analyzed within the merchantable part by its wood and bark portions, and as wood plus bark. All these components of the stem can be observed in Figure 1, where the description of each is as follows:

a - wood of merchantable portion of stem including stump wood,
b - bark of merchantable portion of stem including stump bark,
c - wood plus bark of top portion of stem,
d - wood of merchantable portion of stem excluding stump wood,
e - bark of merchantable portion of stem excluding stump bark,
f - wood of stump portion of stem,
g - bark of stump portion of stem.

Total stem wood and bark, therefore are made up of a, b and c.

Rationale

The ovendry mass of these three stem components, the above mentioned a, b and c, could be analyzed in either of the following two ways: (1) by using the ovendry mass values of components, or (2) by using the percentage values of components. The first approach provides direct ovendry mass (OM) estimates of components; however, the sum of the component estimates will not equal the ovendry mass of the total stem if the latter has been calculated from equations formulated in independent studies. The second method, on the other hand, will ensure that component estimates, once their proportions have been established, will add up to total stem ovendry mass regardless of how the latter was derived. Since this total ovendry mass will often be the only biomass value available, the percentage approach was chosen for the analysis. That is, ovendry mass of merchantable wood, merchantable bark, and top wood plus bark were expressed as percent of ovendry mass of stem wood plus bark (OM%), totaling 100%, and entered into the analysis in that way. Similarly, in the stump analysis, the percentage values were used instead of actual mass values. The percentage approach was also followed in the old Form-class Volume Tables (Forest Service 1930) and later by several other researchers for predicting diameters along the stem, and favored by Honer (1965) for merchantable volume estimations. It was also used by Honer and Heger (1971) in examining the distribution of mass and volume over the tree stem.

For the sum of the component estimates to equal 100% (the total stem estimate), the equations of components to be developed should be based on and contain the same variables. In the estimation of the OM% of merchantable wood, one of these variables would have to be the merchantable top diameter (or merchantable height). However, as the OM% of stump wood plus bark of a given tree at a given stump height is constant and does not vary with the change of the location of the merchantable top diameter, its estimation cannot be related to the same predictors. For this reason, stump had to be studied separately from the aforementioned three stem components.

After the percentage approach is chosen , the mass percentages were tried to be related to the ratio of merchantable top diameter to breast height diameter (also called relative diameter, dm/d). The logic behind this, is deduced from the following argument:

If we assume that all the stems are in the same shape and are proportionally similar in all dimensions (Figure 2), and if

$$\frac{DE}{AB} = \frac{GH}{AF} = \frac{h_2}{h_1} = \frac{h_4}{h_3}, \quad \text{then}$$

$$\frac{\text{area DEC}}{\text{area ABC}} = \frac{\text{area GHI}}{\text{area AFI}}.$$

In the same way, by rotation of these areas around their vertical axes (Alemdag 1978), the ratios of the volumes so created are also equal. Thus, when the ratio of top diameter to bottom diameter remains the same, regardless of the tree size, the ratio of volume above (or below) the top diameter to the whole stem volume will remain the same. Consequently, ovendry mass, which is a product of volume and wood density, will show the same ratio.

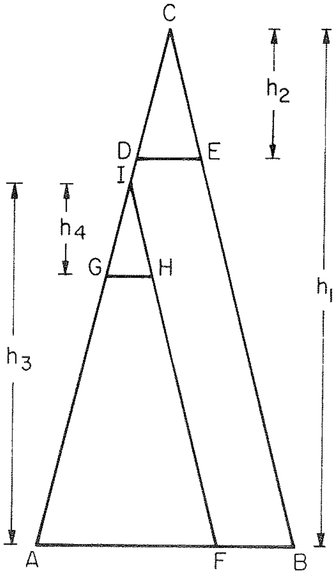

Figure 2. Diagram illustrating the diameter-ratio concept.

Distribution of OM percent of merchantable wood over the dm/d ratio revealed this hypothesis (Figure 3). This illustrated relationship indicated that a given ratio of dm/d corresponds to a given ratio of ovendry mass of merchantable part/ovendry mass of whole stem. It also strongly suggested a function of a second-degree or a third-degree polynomial curve.

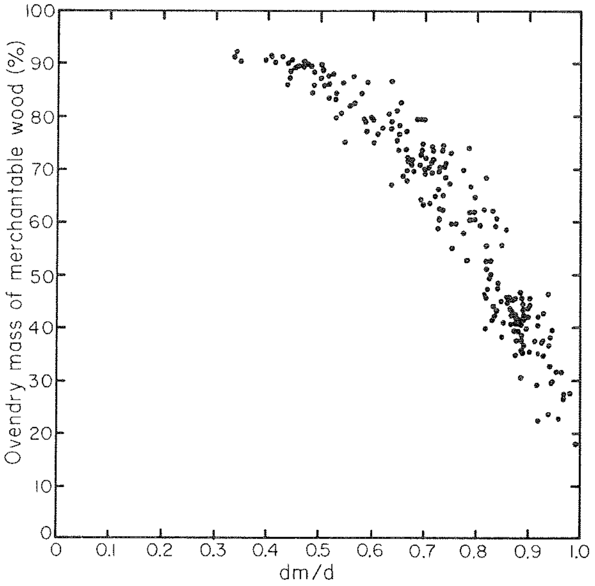

Figure 3. Distribution of observed ovendry mass percents of merchantable wood over merchantable top diameter/breast height diameter ratio in jack pine.

METHODS AND MATERIALS

Data

The sample tree data used for the analysis in this study are those used previously in developing single-tree biomass equations for the above mentioned species in Ontario (Alemdag 1981 and 1982; Alemdag and Horton 1981; Alemdag and Stiell 1982). The red pine data were collected in plantations, all others in natural stands. For each sample tree, the data contained the following variables:

(1) diameter at breast height outside bark (d),
(2) total tree height (h),
(3) merchantable top diameter outside bark (dm) at 1/3, 2/3 and 3/3 of the height at which a diameter of 9.1 cm occurred,
(4) merchantable height (hm) (height from ground level to the merchantable top diameter),
(5) ovendry mass of total stem wood, total stem bark, wood below a given merchantable top diameter, bark below a given merchantable top diameter, wood above a given merchantable top diameter, bark above a given merchantable top diameter, stump wood to a 30-cm stump height and stump bark to a 30-cm stump height. Diameters were recorded in centimetres, heights in metres and mass in kilograms. The smallest merchantable top diameter measured was 9.1 cm with a few exceptions of 8.0 cm. Some of these variables are provided in Table 1.*

Methods

The analyses first centered on merchantable top diameter or its various forms for expressing merchantable wood and the other components. Later on, height to the merchantable top diameter was also used in developing a separate set of estimation equations.

The estimation equations were developed by using the regression analysis technique. Before forming the regression models, two tree species, a softwood (jack pine) and a hardwood (white birch), were chosen as test species. The white birch test data covering a particular locality (the research forest of the Petawawa National Forestry Institute) were drawn from the whole white birch data. After examining several plottings of variables, fourteen test models were prepared for the estimation of OM% of merchantable wood, merchantable bark, and top wood plus bark, and one test model for stump wood plus bark. These models are shown in Table 2. The first eight models are based on breast-height diameter, tree height and merchantable top diameter, the following two models on merchantable top diameter alone, and the next four on breast-height diameter, tree height and merchantable height. The stump model is based on breast-height diameter and tree height, since stump height could not be included in the model because of its fixed value.

Even though the Models 4-8 with dm/d looked most promising, the other models with d, dm and h were also worth testing. Models identical to Model 6 (but without h) and Model 11 were also used by Honer (1965) in his merchantable volume expressions. Note that, theoretically and in practice, the ratio of dm/d is always smaller than 1.0.

As mentioned earlier, the data used in the present study contains ovendry mass values of stump at a constant height of 30 cm. In order to produce estimates of stump percentages at different stump levels, a geometrical analysis was conducted on the stump, considering it as the frustum of a neiloid, and using observed diameters at ground level and at stump height as well as estimated diameters at any given stump level (Alemdag and Honer 1977).

RESULTS AND DISCUSSIONS

Test Results

Results of the regression analysis of the fifteen models using the data of the two test species are summarized in Table 3 for merchantable wood and for stump wood plus bark by their multiple correlation coefficients (R^2) and standard error of estimates as percent of the mean (SEE%). It will be seen that for both species, the first three models performed poorly in estimating merchantable wood by merchantable top diameter. Models 9 and 10 were even worse. The best results were obtained by Models 4, 5, 6, 7 and 8, containing a ratio-variable in the form of dm/d. In all cases this variable or its

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*all the tables are in the appendix.

square or cube form accounted for most of the variation (over 90%) in OM% of merchantable wood. The introduction of tree height to these equations did not make any considerable improvement over dm/d in estimating OM%. In some cases the contribution of the height variable turned out to be not significant at all. This was expected because of the above stated rationale on which these models were based.

Because these four models provide almost the same high degree of correlation and precision as measured by R^2 and SEE%, any one of them would equally be suitable as a final model. The best one among them appeared to be Model 4, and it was adopted for all the species and species combinations. Thus:

$$OM\% = b_0 + b_1 \cdot (dm/d) + b_2 \cdot (dm/d)^2$$
(Model 4)

After it was chosen, this model was checked for the homogeneity-of-variance of the residuals using the OM% and OM of merchantable wood of one of the test species. Either in the percentage or in the absolute value, the residuals were evenly distributed along the zero line, and the estimated values over observed values were closely gathered along the 45° line (Figure 4). These conditions indicated that the equations of this model fit the data quite well and no substantial bias was involved.

Since the merchantable portion of a stem can also be defined by its height instead of by its top diameter, it could also be estimated using its merchantable height. The four test models for the estimation of the three components employing the ratio of merchantable height to total tree height (also called relative height, hm/h), which were based on the same argument stated above, gave distinctively different results, with Model 11 being the best. The breast-height diameter, a variable in these three models, made no improvement over hm/h

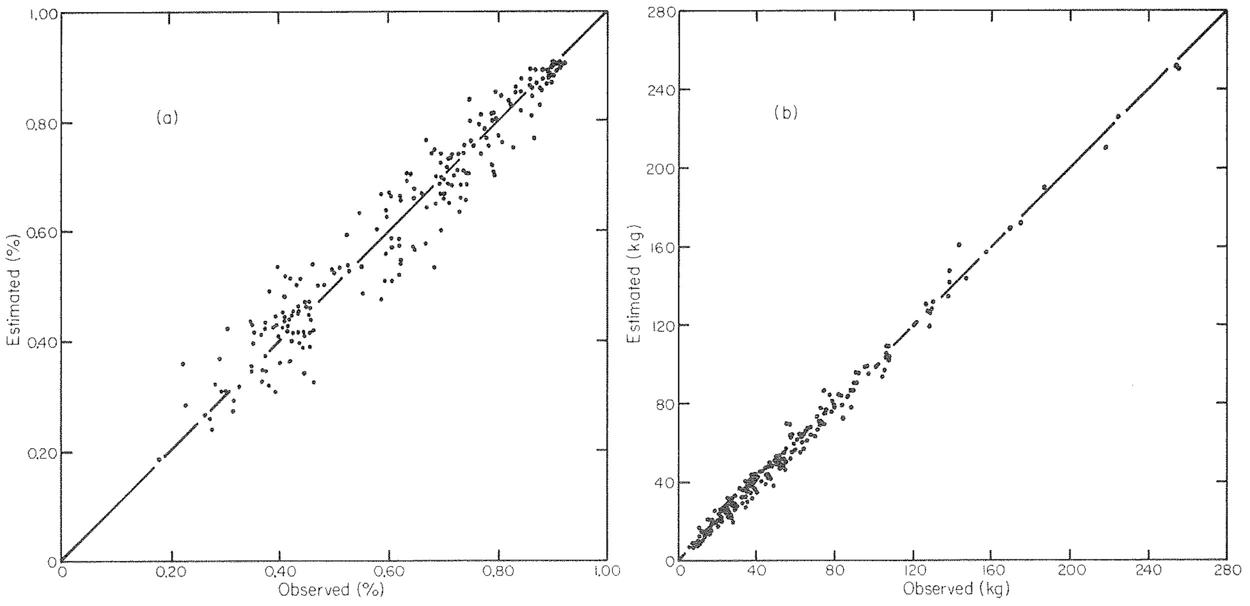


Figure 4. On jack pine, using Model 4, distribution of estimated values along 45° line (a) in terms of OM% and (b) in terms of OM.

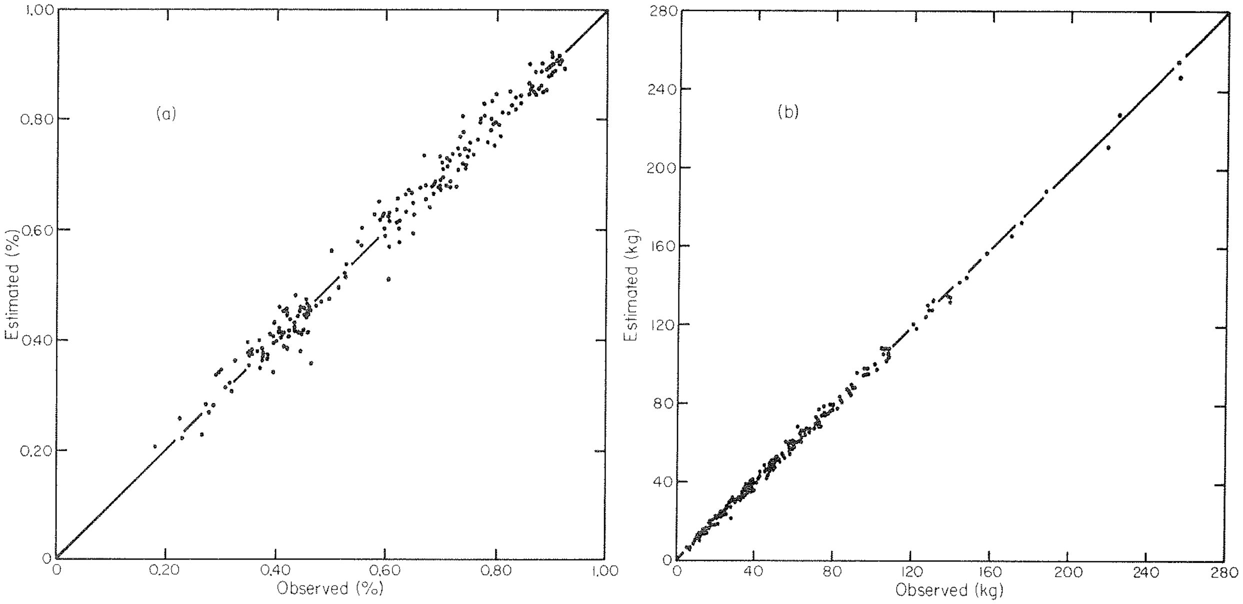


Figure 5. On jack pine, using Model 11, distribution of estimated values along 45° line (a) in terms of OM% and (b) in terms of OM.

or its square or cube in the prediction of OM%. In most of the cases the variable hm/h itself accounted for over 90% of the variation. Therefore, Model 11 was chosen for all the species and the species combinations. Thus:

$$OM\% = b_0 + b_1 \cdot (hm/h) + b_2 \cdot (hm/h)^2 \quad \text{(Model 11)}$$

Here too, the requirement for the homogenous variance of residuals was satisfied, and the fit of the model to the data was acceptable (Figure 5).

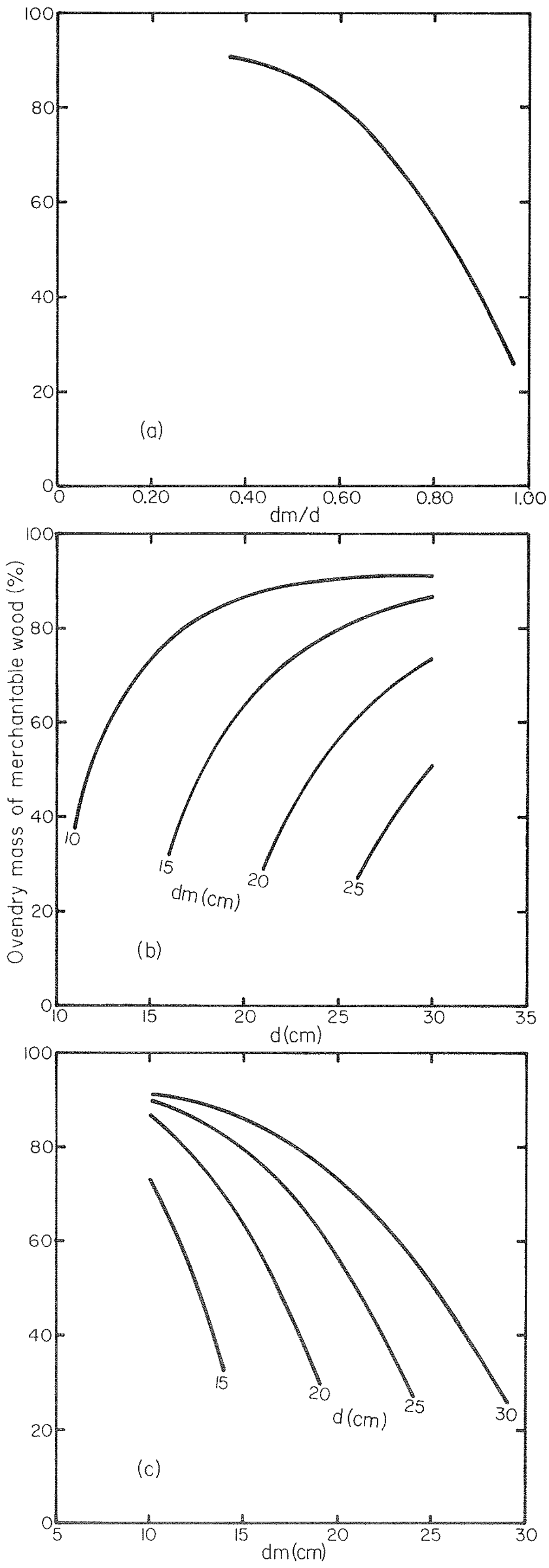
Tests indicated that, owing to insufficient data at the lower end of the dm/d ratios, Model 4 is restricted to dm/d ratios over a given value. In the same manner, Model 11 is not applicable for the hm/h ratios near to 1.0, its highest value. One way of getting around this situation, that is, of covering the full range of mass percentages from 0.0 to 100.0 is to develop conditional equations. While working with relative diameters we know that when dm/d = 0.0, OM% of merchantable wood plus bark is 100.0 and OM% of top wood plus bark is zero. While working with relative heights, when hm/h = 0.0, OM% of merchantable wood plus bark is zero and OM% of top wood plus bark is 100.0; and when hm/h = 1.0, OM% of merchantable wood plus bark is 100.0 and OM% of top wood plus bark is zero. There is no problem in satisfying these conditions by modifying the forms of the models 4 and 11 for the merchantable wood plus bark and for the top wood plus bark, and then separating the foregoing into wood and bark. These were tested but then it was found that, with the form of the equations adopted, there is still one constraint which is not easy to satisfy. This is to ensure that OM% does not go above 100% or below 0% at any point. This would require some restrictions on the values of the b parameters which could not be handled by an ordinary linear regression program. For this reason and because the unconditional equations provided an adequate fit to the test data within the range of interest, further work was not undertaken on this matter.

Although neither of the models 9 and 10 has an acceptably high performance, the better of these two is Model 9. Since this type of an equation has a place of use in practice, this model was adopted for all the species and species combinations, where only the stem mass is available and the dimensional single tree data are lacking.

The only possible model for stump wood plus bark produced a relatively reasonable result with the test species jack pine, but a poor result with white birch. For this reason, Model 15 was not adopted for stump wood plus bark estimations. Instead, average stump mass values of wood and of bark as percent of total stem mass wood plus bark were preferred for all species and species combinations.

Two-dimension equations

Model 4, based on ratio dm/d, was used for developing the equations for predicting percents of merchantable wood, merchantable bark, and tree top wood plus bark where merchantability was defined by the merchantable top diameter. The regression coefficients together with R^2 and SEE% values are provided in Tables 4a-4c for individual species and species combinations (because red pine was from plantations, it was not combined with the other species from natural stands). The best estimates are for merchantable wood percent (R^2 around 0.910 to 0.930 and SEE% approximately 8 to 10%) and top wood plus bark percent. Merchantable bark percent estimates are also good and acceptable. It should be noted that these estimates for a given tree by its d and dm add up to 100%. The behaviour of Model 4 can best be observed by Figure 6, using jack pine merchantable



Opposite column:
Figure 6. Behavior of ovendry mass of merchantable wood percent in jack pine: (a) over dm/d ratio, (b) over d for given dm values, and (c) over dm for given d values.

wood as an example. It will be seen that OM of merchantable wood as percent of OM of stem wood plus bark has either decreasingly or increasingly convex shaped curves over dm/d, d or dm. Model 11 was used for developing the equations where merchantability was defined by the merchantable height. The regression coefficients are given in Tables 5a-5c. The R^2 and SEE% values indicate that these estimation equations based on hm are even better than those based on dm. Here too, estimates add up to 100%.

As it will be seen in Tables 4c and 5c, the R^2 and SEE% values of combined species are as good as those for single species. At the same time their regression coefficients are very similar to those of single species and are consistent in regard to plus or minus signs. For these reasons, using the equations of combined species instead of the individual species could be considered as valid and accurate as the others. This decision is left to the user's judgement regarding the nature of the data and the required precision of the work. In Honer's report (1965) on merchantable volume estimations these equations were recommended only for the combined species since it was found that there was almost no difference between these and the individual species equations in their application.

One-dimension equations

The equations of Model 9, using variable dm only, were developed for estimating the percentage values of merchantable wood, merchantable bark, and top. The regression coefficients for all species and species combinations can be found in Tables 6a-6c. The estimated percentages for a given tree total 100%. The very poor R^2 and SEE% values indicate that little confidence could be placed in estimates obtained with these equations.

Stump deductions

The average stump wood and stump bark values of OM% were calculated as arithmetic means of the data at 30-cm stump height as shown in Table 7. However, since stump values at different stump heights are required in practice, the percentage values given in Table 7 were converted to percentages for different stump heights. First, each commonly needed portion of stump (in volume) from the ground level up to a given height was calculated as percent of stump at 30-cm height (in volume), and it was found that the differences are negligible between the species as well as between the stump sizes. Therefore, a single set of values of these proportions was adopted as shown in Table 8. These values are to be applied to stump wood and stump bark percentages of each species and species combinations provided in Table 7 in order to arrive at stump deduction percentages of wood and bark at various stump heights. An example is provided in Table 9, using jack pine.

APPLICATIONS

The equations of Model 4 are to be used with single trees of a known breast-height diameter and a merchantable top diameter in estimating percentages of the three components of the ovendry mass of stem wood plus bark, i.e., (a) merchantable wood, (b) merchantable bark, and (c) top wood plus bark. These equations can also be used with mean stand diameter, and are still valid if only the ratio dm/d is given, without dm and d being individually specified. The equations of Model 11 are to be applied for the same purposes where total tree height together with the height to the merchantable top diameter, or simply the hm/h ratio is provided. However, since all these equations yield percentage values, they must be applied to the ovendry mass of the stem of a given tree in order to come up with the estimates of ovendry mass of these three components.

The equations of models 4 and 11 have some restrictions: the equations of Model 4 are only applicable for dm/d ratios larger than the ones provided in Table 10, the largest being near to but smaller than 1.0, and the equations of Model 11 are restricted to hm/h ratios smaller than the values given in the same table, the smallest being near to but bigger than zero.

Because the stump was included in the merchantable portion of stem (Figure 1) in developing the equations, the calculated merchantable wood and merchantable bark percentages have to be reduced as much as the stump percentages in order to arrive at net merchantable wood and merchantable bark percentages above a given stump height.

The following is an example for the whole procedure using Model 4.

Let us consider a jack pine stem with d = 12.3 cm, h = 14.00 m and dm = 8.0 cm, cut at a stump height of 20 cm. In this case, the ratio of dm/d is equal to 0.65. When the equations are used with these figures they give 75.46% for merchantable wood, 6.39% for merchantable bark, and 18.15% for top wood plus bark. On the other hand, stump deductions for 20-cm stump height are 2.86% for stump wood and 0.57% for stump bark (Table 9). Therefore, when these are substracted from merchantable wood and merchantable bark, respectively, the actual percentages become 72.60% and 5.82%. The percentage distribution of all these five components of stem wood plus bark can be seen in Table 11 together with some other examples of dm/d for jack pine.

After computing these percentages, actual ovendry mass of the components can easily be calculated by employing them with the ovendry mass of stem wood plus bark of the same tree. By the formula for jack pine previously developed by Alemdag (1982), this mass value is 36.3 kg. Thus, for dm/d = 0.65, in this tree, stump wood becomes 1.0 kg, stump bark 0.2 kg, merchantable wood 26.4 kg, merchantable bark 2.1 kg, and top wood plus bark 6.6 kg. That means, if only the merchantable wood is removed from the stem of this tree, 9.9 kg of stem residue will remain. For the total tree residue, 1.8 kg of live branches and 2.2 kg of twigs plus needles should be added.

Similarly, when two 4-m logs are to be removed from this stem as a merchantable part, then, with hm/h = 8.20/14.00 = 0.59 in Model 11 and with stump deductions taken into consideration, the percentage distribution will be as follows: stump wood 2.86, stump bark 0.57, merchantable wood 75.82, merchantable bark 5.99, and top wood plus bark 14.76.

The equations of Model 9 are solely for the cases where single tree or average stand diameters are not available, and the total stem mass, particularly on an area basis, is derived from volumetric data. If the merchantable top diameter is known, then the component percentages will be calculated and applied to the stand total stem mass to come up with the actual mass values. The stump provisions are to be applied here as well, as explained above.

SUMMARY

1. In developing the equations for estimating the ovendry mass of (a) wood of the merchantable portion of a stem, (b) bark of the merchantable portion of a stem, and (c) wood plus bark of the top of a stem, the percentage approach was preferred over the direct approach.
2. Equations of Model 4 are to be used by dm and equations of Model 11 by hm where dimensional single tree data are available. These equations can also be applied to the mean tree of a stand.
3. Equations of Model 9 are to be used where only total ovendry mass data are available.
4. Stump deductions have to be taken into consideration in order to compute net merchantable-wood and merchantable-bark percentages.
5. Equations of models 4, 11 and 9, and stump deductions can be utilized, either with individual tree species or with the tree species combinations, according to the choice of the user.
6. Equations are applicable only where ovendry mass of stem wood plus bark is available by individual trees (Models 4 and 11) or as a total (Model 9) such as by a given area, by a car load, on a hectare or in a mill yard.
7. The ovendry mass of the merchantable

portion, the top and the stump of a stem would be estimated within the errors associated both with the stem-ovendry mass, and the ovendry mass percentage estimates developed herein.

8. Since it was assumed that the wood density remains the same for a given tree species in a given region, the equations developed herewith to estimate the percent of the ovendry mass of merchantable wood can also be used to approximately estimate the percent of the wood volume that is in the merchantable portion of the stem. In the present case this volume percent is the ratio of the merchantable volume of the stem (inside bark, including stump) to the stem volume (outside bark, including stump and top).

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APPENDIX:

TABLES

Table 1. Statistical data for species and species combinations

Species	Number of trees	Number of observations	d(cm)		h(m)		dm/d		hm/h	
			Mean	Range	Mean	Range	Mean	Range	Mean	Range
Red pine	149	447	22.0	10.5-36.2	17.88	11.76-25.25	0.683	0.251-0.993	0.486	0.143-0.980
Jack pine	72	216	16.6	10.2-26.8	17.81	11.90-23.50	0.735	0.336-0.990	0.429	0.088-0.847
Black spruce	42	126	13.6	10.0-22.2	13.54	9.10-18.90	0.818	0.405-0.991	0.352	0.079-0.773
White spruce	58	174	16.8	10.4-35.8	13.93	6.20-23.20	0.755	0.251-0.991	0.413	0.096-0.862
Balsam fir	46	138	15.1	10.0-27.4	14.54	8.00-19.20	0.784	0.329-0.992	0.384	0.081-0.781
Trembling aspen	164	492	19.5	10.1-43.5	19.53	9.58-27.25	0.731	0.209-0.995	0.422	0.083-0.852
Largetooth aspen	71	213	19.2	9.6-39.2	19.71	11.60-28.90	0.716	0.232-0.990	0.429	0.066-0.849
White birch	103	309	19.4	10.0-32.7	18.21	11.70-22.25	0.716	0.278-0.992	0.426	0.071-0.797
Red oak	36	108	25.7	16.4-40.4	18.88	14.25-23.00	0.650	0.223-0.976	0.489	0.214-0.831
All softwoods*	218	654	15.8	10.0-35.8	15.27	6.20-23.50	0.767	0.251-0.992	0.400	0.079-0.862
All hardwoods	374	1 122	20.0	9.6-43.5	19.14	9.58-28.90	0.716	0.209-0.995	0.431	0.066-0.852
All species*	592	1 776	18.4	9.6-43.5	17.71	6.20-28.90	0.735	0.209-0.995	0.420	0.079-0.862

*Excluding red pine.

Table 2. Regression models tested

Model No.	Model form
	For merchantable wood, merchantable bark, and tree top wood plus bark by dm
1	$OM\% = b_0 + b_1 \cdot dm^2 + b_2 \cdot d^2 + b_3 \cdot h$
2	$OM\% = b_0 + b_1 \cdot dm^2 + b_2 \cdot h(dm^2)^2 + b_3 \cdot d^2(dm^2)^2$
3	$OM\% = b_0 + b_1 \cdot dm^2 + b_2 \cdot (dm^2)^2 + b_3 \cdot h \cdot dm^2 + b_4 \cdot d^2 \cdot dm^2$
4	$OM\% = b_0 + b_1 \cdot (dm/d) + b_2 \cdot (dm/d)^2$
5	$OM\% = b_0 + b_1 \cdot (dm/d) + b_2 \cdot (dm/d)^2 + b_3 \cdot h$
6	$OM\% = b_0 + b_1 \cdot (dm^2/d^2) + b_2 \cdot (dm^2/d^2)^2 + b_3 \cdot h$
7	$OM\% = b_0 + b_1 \cdot (dm/d)^2 + b_2 \cdot h$
8	$OM\% = b_0 + b_1 \cdot (dm/d)^3 + b_2 \cdot h$
9	$OM\% = b_0 + b_1 \cdot dm$
10	$OM\% = b_0 + b_1 \cdot dm^2$
	For merchantable wood, merchantable bark, and tree top wood plus bark by hm
11	$OM\% = b_0 + b_1 \cdot (hm/h) + b_2 \cdot (hm/h)^2$
12	$OM\% = b_0 + b_1 \cdot (hm/h) + b_2 \cdot (hm/h)^2 + b_3 \cdot d$
13	$OM\% = b_0 + b_1 \cdot (hm/h)^2 + b_2 \cdot d$
14	$OM\% = b_0 + b_1 \cdot (hm/h)^3 + b_2 \cdot d$
	For stump wood plus bark
15	$OM\% = b_0 + b_1 \cdot d + b_2 \cdot d^2 + b_3 \cdot h$

Table 3. **Statistical results for the models applied to two test species**

Model No.		Jack pine		White birch	
		R^2	SEE%	R^2	SEE%
	(a)	For merchantable wood by dm			
1		0.715	17.11*	0.827	11.41
2		0.363	25.58	0.768	13.21
3		0.680	18.17	0.915	8.07
4		0.931	8.39	0.947	6.31
5		0.931	8.40	0.947	6.34
6		0.930	8.46	0.945	6.43
7		0.919	9.10	0.924	7.54
8		0.931	8.42	0.944	6.45
9		0.263	27.39	0.671	15.59
10		0.218	28.22	0.620	16.76
	(b)	For merchantable wood by hm			
11		0.981	4.36	0.988	2.97
12		0.982	4.27	0.989	2.84
13		0.865	11.76	0.873	9.74
14		0.751	15.97	0.776	12.93
	(c)	For stump wood plus bark			
15		0.683	9.24	0.249	19.47
number of trees		72		35	
number of observations		216		105	
d range, cm		10.2 - 26.8		16.3 - 29.7	
h range, m		11.90 - 23.50		16.48 - 22.25	
dm/d range		0.336 - 0.990		0.303 - 0.964	
hm/h range		0.088 - 0.847		0.187 - 0.782	

*In this and the following tables this value is shown like: (standard error of estimate/mean)·100 = (0.104887/0.613116)·100 = 17.11.

Table 4a. **Regression coefficients for Model 4 for the softwood species**

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
			Red pine		
Merchantable wood	70.523	109.966	-157.566	0.917	8.44
Merchantable bark	7.474	8.754	-13.608	0.712	16.80
Top wood plus bark	22.003	-118.720	171.174	0.916	22.18
			Jack pine		
Merchantable wood	69.383	120.937	-171.664	0.931	8.40
Merchantable bark	4.143	13.336	-15.203	0.630	18.25
Top wood plus bark	26.474	-134.273	186.867	0.933	16.22
			Black spruce		
Merchantable wood	40.410	191.766	-209.326	0.918	10.06
Merchantable bark	2.046	24.044	-23.326	0.714	18.41
Top wood plus bark	57.544	-215.810	232.652	0.919	14.27
			White spruce		
Merchantable wood	70.282	103.401	-148.077	0.948	7.20
Merchantable bark	5.835	17.202	-19.792	0.634	20.64
Top wood plus bark	23.883	-120.603	167.869	0.946	14.41
			Balsam fir		
Merchantable wood	46.841	160.823	-185.023	0.909	10.00
Merchantable bark	11.463	11.381	-19.944	0.780	19.22
Top wood plus bark	41.696	-172.204	204.967	0.922	15.55

Table 4b. Regression coefficients for Model 4 for the hardwood species

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
			Trembling aspen		
Merchantable wood	63.449	102.391	-144.963	0.894	11.18
Merchantable bark	16.920	5.385	-18.227	0.660	25.24
Top wood plus bark	19.631	-107.776	163.190	0.911	19.86
			Largetooth aspen		
Merchantable wood	67.474	82.794	-131.142	0.902	10.31
Merchantable bark	14.522	20.714	-31.798	0.730	20.65
Top wood plus bark	18.004	-103.508	162.940	0.925	18.35
			White birch		
Merchantable wood	59.476	121.358	-159.218	0.901	9.94
Merchantable bark	13.736	9.510	-19.791	0.551	29.91
Top wood plus bark	26.788	-130.868	179.009	0.912	20.88
			Red oak		
Merchantable wood	71.810	64.652	-107.916	0.909	8.21
Merchantable bark	10.533	28.059	-33.460	0.619	20.08
Top wood plus bark	17.657	-92.711	141.376	0.913	24.64

Table 4c. Regression coefficients for Model 4 for the combined species

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
			Softwoods*		
Merchantable wood	66.162	118.900	-162.050	0.923	9.13
Merchantable bark	6.056	14.232	-17.350	0.504	27.17
Top wood plus bark	27.782	-133.132	179.400	0.925	15.99
			Hardwoods		
Merchantable wood	64.162	99.767	-142.542	0.892	10.74
Merchantable bark	14.999	11.394	-22.501	0.618	26.31
Top wood plus bark	20.839	-111.161	165.043	0.911	20.61
			Softwoods and hardwoods*		
Merchantable wood	60.523	118.420	-157.064	0.888	10.97
Merchantable bark	17.070	-1.943	-11.349	0.505	34.50
Top wood plus bark	22.407	-116.477	168.413	0.916	18.90

*Excluding red pine.

Table 5a. Regression coefficients for Model 11 for the softwood species

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
Red pine					
Merchantable wood	2.195	187.757	-98.499	0.976	4.48
Merchantable bark	0.150	19.840	-11.524	0.756	15.46
Top wood plus bark	97.655	-207.597	110.023	0.975	12.03
Jack pine					
Merchantable wood	5.985	172.893	-84.215	0.981	4.36
Merchantable bark	1.266	13.730	-8.046	0.662	17.44
Top wood plus bark	92.749	-186.623	92.261	0.983	8.09
Black spruce					
Merchantable wood	6.454	168.476	-80.093	0.951	7.77
Merchantable bark	0.831	18.755	-11.461	0.779	16.18
Top wood plus bark	92.715	-187.231	91.554	0.956	10.52
White spruce					
Merchantable wood	6.990	171.956	-88.106	0.975	5.00
Merchantable bark	0.737	22.026	-13.753	0.689	19.05
Top wood plus bark	92.273	-193.982	101.859	0.978	9.21
Balsam fir					
Merchantable wood	6.651	166.650	-87.306	0.954	7.15
Merchantable bark	0.805	21.737	-8.266	0.787	18.92
Top wood plus bark	92.544	-188.387	95.572	0.962	10.78

Table 5b. Regression coefficients for Model 11 for the hardwood species

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
Trembling aspen					
Merchantable wood	0.943	182.017	-101.509	0.948	7.82
Merchantable bark	0.978	27.838	-10.393	0.692	24.02
Top wood plus bark	98.079	-209.855	111.902	0.964	12.51
Largetooth aspen					
Merchantable wood	2.470	170.247	-91.041	0.969	5.79
Merchantable bark	0.525	34.983	-15.863	0.771	19.03
Top wood plus bark	97.005	-205.230	106.904	0.990	6.64
White birch					
Merchantable wood	4.707	178.755	-98.470	0.956	6.58
Merchantable bark	0.935	26.566	-11.562	0.558	29.70
Top wood plus bark	94.358	-205.321	110.032	0.962	13.73
Red oak					
Merchantable wood	3.267	179.006	-100.728	0.943	6.51
Merchantable bark	-1.532	47.285	-31.095	0.711	17.50
Top wood plus bark	98.265	-226.291	131.823	0.965	15.54

Table 5c. Regression coefficients for Model 11 for the combined species

Component	Regression coefficients				
	b_0	b_1	b_2	R^2	SEE%
			Softwoods*		
Merchantable wood	6.702	168.808	-82.835	0.966	6.06
Merchantable bark	0.688	20.446	-12.905	0.556	25.70
Top wood plus bark	92.610	-189.254	95.740	0.972	9.78
			Hardwoods		
Merchantable wood	2.200	180.025	-100.098	0.948	7.49
Merchantable bark	0.798	29.775	-12.635	0.653	25.08
Top wood plus bark	97.002	-209.800	112.733	0.967	12.62
			Softwoods and hardwoods*		
Merchantable wood	3.765	177.386	-96.390	0.946	7.61
Merchantable bark	1.116	23.490	-8.537	0.497	34.75
Top wood plus bark	95.119	-200.876	104.927	0.968	11.68

*Excluding red pine.

Table 6a. Regression coefficients for Model 9 for the softwood species

Component	Regression coefficients		R^2	SEE%
	b_0	b_1		
		Red pine		
Merchantable wood	96.679	-2.114	0.338	23.8
Merchantable bark	9.563	-0.206	0.281	26.5
Top wood plus bark	-6.242	2.320	0.340	62.0
		Jack pine		
Merchantable wood	101.537	-3.370	0.263	27.4
Merchantable bark	8.751	-0.284	0.279	25.4
Top wood plus bark	-10.288	3.654	0.272	53.4
		Black spruce		
Merchantable wood	90.140	-3.359	0.149	32.4
Merchantable bark	11.284	-0.512	0.320	28.3
Top wood plus bark	-1.424	3.871	0.166	45.5
		White spruce		
Merchantable wood	84.180	-2.001	0.146	29.1
Merchantable bark	11.480	-0.368	0.310	28.3
Top wood plus bark	4.340	2.369	0.168	56.5
		Balsam fir		
Merchantable wood	87.117	-2.798	0.193	29.7
Merchantable bark	11.992	-0.376	0.117	38.4
Top wood plus bark	0.891	3.174	0.188	50.0

Table 6b. Regression coefficients for Model 9 for the hardwood species

Component	Regression coefficients		R^2	SEE%
	b_0	b_1		
		Trembling aspen		
Merchantable wood	73.223	-1.303	0.128	32.0
Merchantable bark	13.849	-0.250	0.083	41.4
Top wood plus bark	12.928	1.553	0.128	61.9
		Largetooth aspen		
Merchantable wood	77.509	-1.691	0.179	29.8
Merchantable bark	17.429	-0.411	0.154	36.5
Top wood plus bark	5.062	2.102	0.186	60.4
		White birch		
Merchantable wood	90.645	-2.330	0.272	26.9
Merchantable bark	15.004	-0.392	0.143	41.3
Top wood plus bark	-5.649	2.722	0.270	60.1
		Red oak		
Merchantable wood	91.861	-1.795	0.491	19.4
Merchantable bark	20.653	-0.476	0.574	21.1
Top wood plus bark	-12.514	2.271	0.547	55.9

Table 6c. Regression coefficients for Model 9 for the combined species

Component	Regression coefficients		R^2	SEE%
	b_0	b_1		
		Softwoods*		
Merchantable wood	87.694	-2.530	0.160	30.2
Merchantable bark	10.318	-0.339	0.176	35.0
Top wood plus bark	1.988	2.869	0.172	53.1
		Hardwoods		
Merchantable wood	78.364	-1.543	0.174	29.7
Merchantable bark	15.062	-0.311	0.117	40.0
Top wood plus bark	6.574	1.854	0.177	62.7
		Softwoods and hardwoods*		
Merchantable wood	78.945	-1.652	0.157	30.1
Merchantable bark	11.714	-0.198	0.040	48.0
Top wood plus bark	9.341	1.850	0.146	60.2

*Excluding red pine.

Table 7. Average stump values at 30-cm stump height, as percent of the total stem mass

Species	Mean value of			Stump wood plus bark's	
	Stump wood	Stump bark	Stump wood plus bark	SD†	SE†
Red pine	4.41	0.77	5.18	1.123	0.053
Jack pine	4.19	0.83	5.02	0.802	0.055
Black spruce	5.66	0.78	6.44	1.412	0.126
White spruce	5.90	0.82	6.72	2.110	0.160
Balsam fir	5.03	0.87	5.90	1.874	0.160
Trembling aspen	3.53	0.80	4.33	1.078	0.049
Largetooth aspen	3.34	0.88	4.22	0.862	0.059
White birch	5.16	0.88	6.04	1.573	0.089
Red oak	5.84	1.03	6.87	1.244	0.120
Softwoods*	5.10	0.83	5.93	1.734	0.068
Hardwoods	4.17	0.86	5.03	1.556	0.465
Softwoods and hardwoods*	4.51	0.85	5.36	1.681	0.040

*Excluding red pine.

†SD = standard deviation; SE = standard error of the mean.

Table 8. Volume (and mass) percentages at different stump heights in relation to stump volume at 30 cm

Stump height (cm)	%
5	17.95
10	35.28
15	52.07
20	68.36
25	84.45
30	100.00

Table 9. Deduction percentages of stump wood mass and stump bark mass at different stump heights in total stem mass (wood plus bark) for jack pine

Stump height (cm)	Stump wood (%)	Stump bark (%)	Stump wood plus bark (%)
5	0.75	0.15	0.90
10	1.48	0.29	1.77
15	2.18	0.43	2.61
20	2.86	0.57	3.43
25	3.54	0.70	4.24
30	4.19	0.83	5.02

Table 10. Permissable ratios for the equations of Model 4 and of Model 11

Tree species	Model 4	Model 11
	Smallest permitted dm/d	Largest permitted hm/h
Red pine	0.347 †	0.894
Jack pine	0.359	0.879
Black spruce	0.464	0.860
White spruce	0.359	0.924
Balsam fir	0.420	0.930
Trembling aspen	0.330	0.885
Largetooth aspen	0.318	0.842
White brich	0.366	0.819
Red oak	0.328	0.858
Softwoods*	0.371	0.891
Hardwoods	0.337	0.857
Softwoods and hardwoods*	0.346	0.859

*Excluding red pine.

† A dm/d ratio of 0.347 means, for example, 7/20.2, 8/23.1, 9/25.9 and 10/28.8, and a value such as 0.100 is not realistic for the species studied.

Table 11. Percentage distribution of stump, merchantable part and top of the stem by various stump heights for jack pine using Model 4

dm/d	Stump height (cm)	Stump wood	Stump bark	Net Merchantable wood	Net Merchantable bark	Top wood plus bark	Total
		% of total stem ovendry mass					
0.40	10	1.48	0.29	88.81	6.76	2.66	100.00
	20	2.86	0.57	87.43	6.48	2.66	100.00
	30	4.19	0.83	86.10	6.22	2.66	100.00
0.65	10	1.48	0.29	73.98	6.10	18.15	100.00
	20	2.86	0.57	72.60	5.82	18.15	100.00
	30	4.19	0.83	71.27	5.56	18.15	100.00
0.90	10	1.48	0.29	37.70	3.54	56.99	100.00
	20	2.86	0.57	36.32	3.26	56.99	100.00
	30	4.19	0.83	34.99	3.00	56.99	100.00